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RESEARCH REPORT CCS-392

# MATHEMATICAL OPTIMIZATION-- A SUCCESSFUL TOOL FOR LOGISTICS PROBLEMS

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# ABSTRACT

Recent developments in mathematical optimization are substantially enhancing the scope and power of logistics planning systems. Based on these advances, successful applications of sophisticated mathematical optimization logistics systems are occurring worldwide. This paper briefly discusses some of these applications and advances.

# 1. INTRODUCTION

Inflationary trends and rising energy costs encountered in many countries have magnified the importance of problems in logistics management. As technological innovation shows itself increasingly limited as a means of holding down the cost of energy, better logistics planning is becoming essential. The complexity of logistical considerations in today's national and multinational business environments makes this no easy task. Accordingly, more and more firms are recognizing the importance of computer-based approaches to logistics planning.

At the same time, new developments in mathematical optimization are greatly enhancing the scope and power of these planning approaches. Less than a decade ago, a far-sighted effort to apply the tools of computer analysis to logistics planning, in any but the most elementary way, was impractical because of the absence of data bases, effective solution methodology, and computer software for these types of planning problems. However, recent innovations in mathematical optimization and computer software design have brought about remarkable successes in solving logistics problems of considerable complexity. By making use of these innovations, millions of dollars are being saved annually by major firms across many different industries. At the same time, customer service is being improved, billing cycle times are being reduced, and operations throughout the corporation are being streamlined, due to the increased efficiencies made possible by the use of sophisticated logistics planning systems. Many of these systems integrate planning and decision-making in the areas of production, inventory, warehouse location and sizing, and marketing.

The key element of many of these advances has been a series of technological breakthroughs in network and discrete optimization techniques that enable large and complex planning problems to be solved routinely and efficiently. The new procedures often require less than one one-hundreth of the computer time and cost of methodologies previously applied. Simultaneously, new advances in conceptualizing and formulating the fundamental decision models underlying logistics problems have greatly improved the usefulness of the solution results. These advances used in concert have shown that mathematical optimization is an extremely valuable tool for determining cost saving insights.

Successful applications of sophisticated mathematical optimization logistics systems are occurring worldwide. At Cahil May Roberts, one of Ireland's largest pharmaceutical companies, a stochastic programming model [27] is determining distribution center locations, service territories, and customer servicing schedules, yielding a 20% annual reduction in delivery and transportation costs. At Kelly-Springfield Tire Company, a subsidiary of Goodyear, a logistics planning system has been developed and installed [31]. Utilizing a sophisticated linear programming/dynamic programming decomposition technique, the system has achieved documented cost savings of over $8 million annually. At International Paper Company an extremely versatile logistics modeling capability has been established by creating a logistics modeling language that can operate in tandem with various optimization packages [10].

At Hunt-Wesson Foods, Inc. (with annual sales of $450 million and growing) a computer-based method was employed not only to resolve imme-

diate distribution center expansion and relocation issues, but also to rebalance distribution center locations, make appropriate customer assignments, and aggregate product flows [15]. Realizable annual cost savings were estimated to be in the low seven figures.

Another attribute which corporations have derived from using these systems is an awareness of the benefits to be gained from integrated planning efforts, i.e., conducting planning across all departments simultaneously in one model. This has caused many companies to create planning committees composed of production, distribution, and marketing managers who use these advanced logistics systems to determine the impact of alternative production schedules and marketing programs on total corporate costs. This usage has greatly improved the communication and coordination between departments because the managers are now able to better understand the needs and problems of their fellow managers.

In this paper we will focus on how advances in two areas, network optimization and integer programming, have made it possible to build new types of logistics models. The power of these models is illustrated by discussing corporate applications of these tools.

## 2. NETWORK OPTIMIZATION

The 1970's were marked by a number of breakthroughs in optimization capabilities for network and network-related problems. Initially, there were specialized primal simplex codes that could solve capacitated transportation and transshipment problems [2, 4, 5, 6, 7, 8, 9, 12, 13, 19, 20, 23, 26, 28, 30, 32, 33, 35, 37, 38, 41]. Later, codes were developed for generalized networks [3, 11, 14, 18, 29, 34, 36] and for linear programming problems with large embedded networks [22]. These codes demonstrated unprecedented efficiency, ranging from

10 to 200 times as fast as the best general purpose linear programming packages (such as MPSX/370 or APEX III) on network oriented problems [16, 17]. As a result, it has become possible to solve huge network problems on a routine basis, opening up a variety of important new applications. An example that illustrates the scope and significance of these applications follows.

## 3. A CHEMICAL FERTILIZER COMPANY APPLICATION

Agrico Chemical Company is one of the largest chemical fertilizer companies in the U.S., with annual sales exceeding half a billion dollars. The web of interacting influences spanning its production, inventory, and distribution segments was perceived by Agrico's management to require an integrated computer-based logistics planning system to aid in making appropriate decisions [24].

A number of key questions needed to be answered. For long-range planning, the sizing and configuration of the distribution system was to be addressed by questions of the form:

- Where should distribution centers be located and what should be their size?
- What long-term inventory investments should be made?
- What transportation equipment investments should be made?
- Which supply, purchase and exchange opportunities should be exploited?

A planning system was designed by Analysis, Research and Computation, Inc., working closely with Agrico's top level management, to answer

these and related questions. The result saved Agrico over $1 million during the first year.

Nevertheless, these savings from long-range planning were only a small part of the total savings made possible by the logistics system once its design was complete and applied on a company-wide basis. Particularly dramatic were the impacts gleaned from using the system for short-range planning decisions. In this mode, the system was used to help answer questions of the form:

- Which products should be produced, and in what quantities?
- Which production centers should be used to produce what product mixes?
- What allocations of the various products should be shipped from which production center to which distribution center?
- Which distribution centers should be used to meet the demands of which customers?
- What modes of transportation should be used?
- What is the best way to time the production, inventory holding, and distribution to respond to seasonal and other fluctuations in demand?

During its year of operation in 1976, the use of the logistics system in short-range planning enabled Agrico's management to reduce distribution costs by $3.7 million. However, initially Agrico could not implement all of the decision alternatives suggested by the logistics system, due to the normal difficulties of rescheduling production and distribution activities. As the system proved itself, and time permitted more of the suggested alternatives to be adopted, the annual dollar savings correspondingly increased. In 1977, 1978, and 1979 these

annual savings amounted to $6.7 million, $7.7 million, and $11.9 million, respectively.

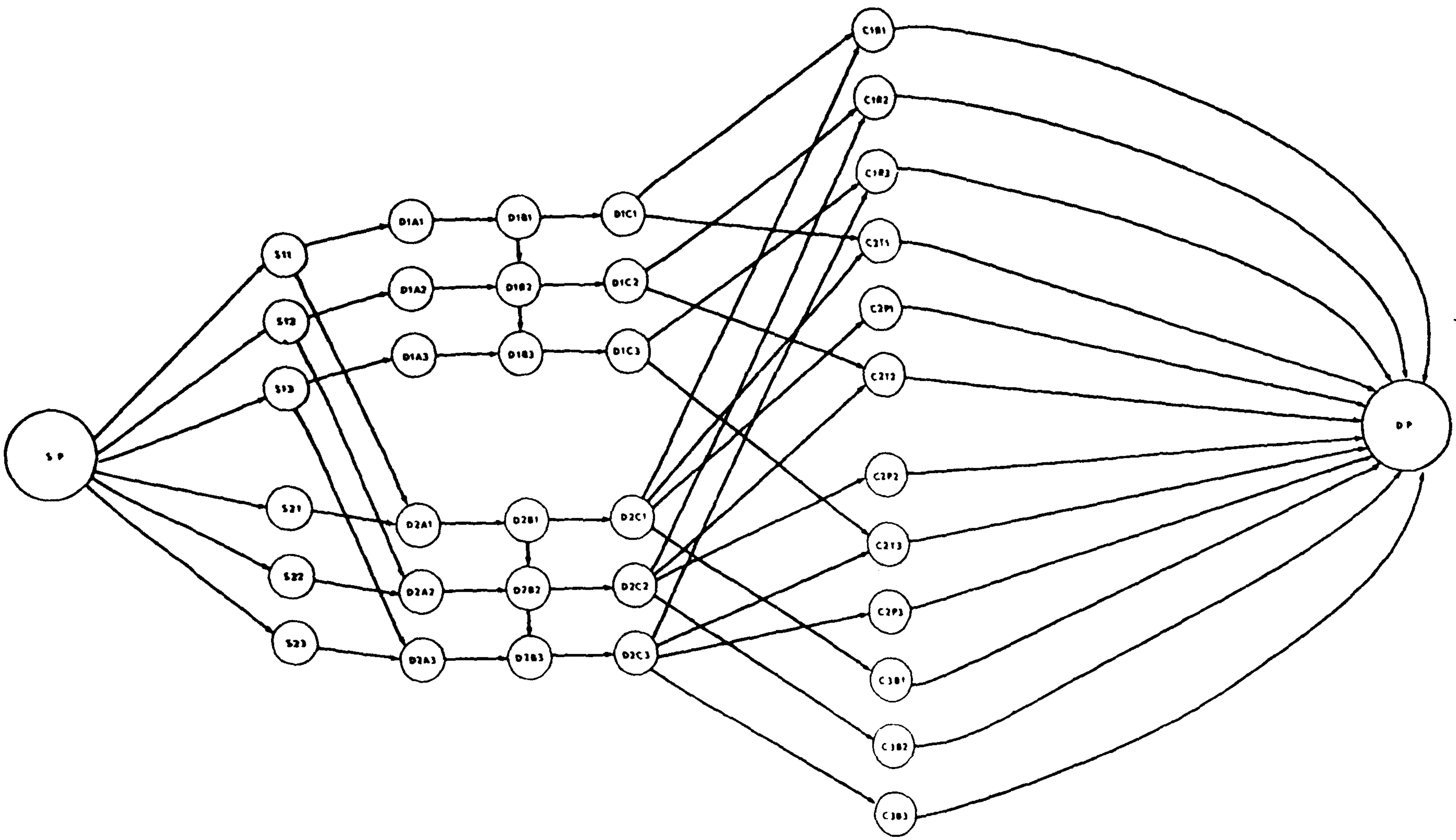
Indirect sources of cost savings are also worth noting. Commitment to the planning system necessitated developing an on-line automated freight rate system. Though initially designed simply to provide data input to the planning system, the automated freight rate system allowed Agrico to invoice a customer at the time an order is placed, resulting in a reduction of average billing time by approximately one day. The corresponding reduction of accounts receivable produced a working capital interest savings of more than $50,000 annually. Moreover, the planning system has enabled Agrico to reduce the number of stockouts suffered by clients by more effectively prepositioning its products in the marketplace, resulting in increased sales and yielding more than $850,000 of working capital annually.

Still another outcome of using the logistics planning system for short-range planning is improved coordination and information flow between departments. In the long run, the impact of improved departmental communication may be as important as the more quantifiable cost reductions.

## 3.1 BASIC MODEL COMPONENT OF AGRICO'S SYSTEM

The interconnections of the basic elements of Agrico's operations, as modeled by the logistics planning system, are usefully displayed in the form of a network diagram as illustrated in Figure 1. The smaller nodes (circles) of this diagram represent production sites, distribution centers and customer demand zones. A master supply policy node and a master demand policy node have also been utilized to depict pro-

FIGURE 1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duction and demand alternatives, as will be described shortly. The arcs (arrows) indicate allowable flows, or movements of goods. Flows on arcs include both movements of a physical type (as in shipping goods from production sites to distribution centers, or from distribution centers to customers) and movements of an assignment type (as in assigning a certain quantity of product to be made at a particular production center). (Conventions regarding network diagrams and their interpretations in general may be found, for example, in [16, 17, 21, 24].)

Figure 1 represents a simplified three-month "snapshot" of the overall model structure. Flows on arcs leading from the master supply policy node designated SP, represent quantities of product made in three different months at each of two different production sites. Production quantities for Site 1 are characterized in these three periods by the flows on the three arcs from SP to the S1t nodes representing Site 1 in time period t, while production quantities for Site 2 are similarly characterized by the flows on the three arcs from SP to the S2t nodes. Though not shown in this diagram, costs and upper and lower bounds are associated with these and all other arcs of the network, identifying the unit cost of flow on each arc and the maximum and minimum admissible quantities of this flow. Thus, for example, costs on arcs from the master supply policy node would identify the cost of producing each unit of product at Production Sites 1 and 2 (in each of the three months under consideration), while bounds on these arcs would identify the lower and upper production limits for the sites.

Arcs from the two groups of production site nodes lead to two groups of distribution center nodes, representing two different distribution centers in the same three periods. (Note in Figure 1 that the first pro-

duction site can ship goods to either of the two distribution centers within the same period, as indicated by the pair of arcs leaving each node for Production Site 1. However, the second production site can ship goods only to the second distribution center.)

The two distribution center node groups are not only subdivided vertically to differentiate time periods, but are also subdivided horizontally to differentiate functions of the distribution process. These functions consist of receiving goods from a production site or another distribution center, holding inventory, and sending goods out of the distribution center to customers or other distribution centers. By this subdivision of nodes, it is possible to associate different arcs, and hence different costs and bounds, with each of the functions. Specifically, the horizontal arc from the DjAt node identifies the flow of goods received at distribution center j in period t, the downward arc from the DjBt distribution center j node identifies the flow of goods held from period t to the next (hence the amount of inventory), while the horizontal arc leading to the DjCt distribution center j node identifies the flow of goods sent out in period t.

The customer demand nodes for customer k are similarly subdivided by time period 5. These nodes are also further subdivided to differentiate demand by mode of transportation m (rail, truck, pipeline, and barge). Arcs from the distribution center outbound freight nodes, DjCt, to the customer demand nodes, Ckmt, therefore represent possible shipment of goods from particular distribution centers to particular customers by specific transportation modes in given periods. Arcs from customer demand nodes to the master demand policy node, designated DP, carry bounds and costs applicable to the total amount of product required by a

customer (for a given transportation mode and time period), independent of which distribution center supplies the product.

The objective of this model may be expressed as minimizing the sum of:

- production costs
- transportation costs to move product to the customer
- inventory holding costs
- distribution center throughput costs,

while simultaneously satisfying:

- demand volume of customers
- supply availability at supply points (or production sites)
- distribution center input-output capacity
- inventory capacity
- opening inventory levels
- minimum closing inventory requirements
- mode of shipment required.

The entire distribution system for Agrico consists of four production sites, 78 distribution centers, and over 5000 total customer demand points (when differentiated by time period and mode of transportation). The network model representation of this entire system typically consists of approximately 6000 nodes and more than 35,000 arcs. Initial attempts to solve this problem (utilizing an existing network code to which Agrico has access) proved disappointing. Solution time was approximately two and a half hours on an Amdahl V-6 computer. However, major recent innovations in network solution technology provided a new advanced and highly efficient code, called ARCNET [24]. This code was able to reduce the solution time from

2-1/2 hours to 50 seconds. Because of the need to solve the model repeatedly in order to answer "what if" questions, this rapid solution time is a vital element to the success of Agrico's logistics planning system.

3.2 EXTENSIONS OF THE AGRICO MODEL

The basic model described above was sufficient to perform the short-range planning analyses discussed previously. However, to answer the long-range planning questions, a more sophisticated extension of the basic model was required. The extended model is a large-scale, mixed integer, linear programming problem whose LP portion involves a large embedded network structure (the basic model). The integer characteristic is required to locate the size distribution facilities in the long-range analysis. The non-network linear constraints are required to model multi-commodity and joint capacity restrictions. Although mathematical procedures for efficiently solving LP problems with embedded networks had been developed, at the time such procedures had never been implemented for use on a computer. Agrico's need led to another major innovation in solution technology [22], the development of the solution code, NETLP, to solve the long-range problem. NETLP, when compared to APEX-III, proved to be at least 75 times faster on medium size prototypes of Agrico's long-range problems. At Agrico now, using NETLP, problems with 23,000 variables and 6250 equations, including a number of non-network constraints, are normally solved in less than three minutes on an Amdahl 470 V-6 computer. Again, this efficient solution capability, in terms of providing management with a practical tool which can be

used on a routine basis to evaluate planning alternatives, is of vital importance.

The significance of the Agrico model is not only that it represents a new type of comprehensive production, distribution, and inventory planning system, but that its success underscores the value of the new network optimization techniques. Similar problems have been attacked using conventional linear programming technology and have been found to be computationally impractical. Similar types of models are being developed in a number of other companies in such industries as aluminum, pesticides, forestry, automobiles, poultry, dairy, and mining.

## 4. INTEGER PROGRAMMING

Another area that has witnessed advances with major consequences for logistics modeling is integer programming. The opinion of many practitioners has been that integer problems of substantial size are unsolvable within practical time limits, assuming one requires a near optimal solution. Recent research, in contrast, has shown that certain classes of integer programming problems can be solved very efficiently. For example, the papers [16, 17, 21, 25] contain descriptions of 0-1 integer problems containing 4000-20000 variables which have been solved in a matter of seconds. Many of these applications use a network modeling approach, called NETFORM, and employ solution codes developed explicitly for the application.

Recent research [40] has shown that an important class of facility location/allocation problems can be modeled as a NETFORM which is amenable to solution. The characteristics of logistics location

and allocation problems which have been handled successfully using this approach are discussed in the following section.

## 5. LOGISTICAL LOCATION AND ALLOCATION PROBLEM CHARACTERISTICS

Logistical location and allocation problems which have been solved successfully primarily involve single time period problems where, given a finite set of possible facility sites and associated capacities, one wants to determine:

i) the set of optimal facility sites and capacities to utilize, and

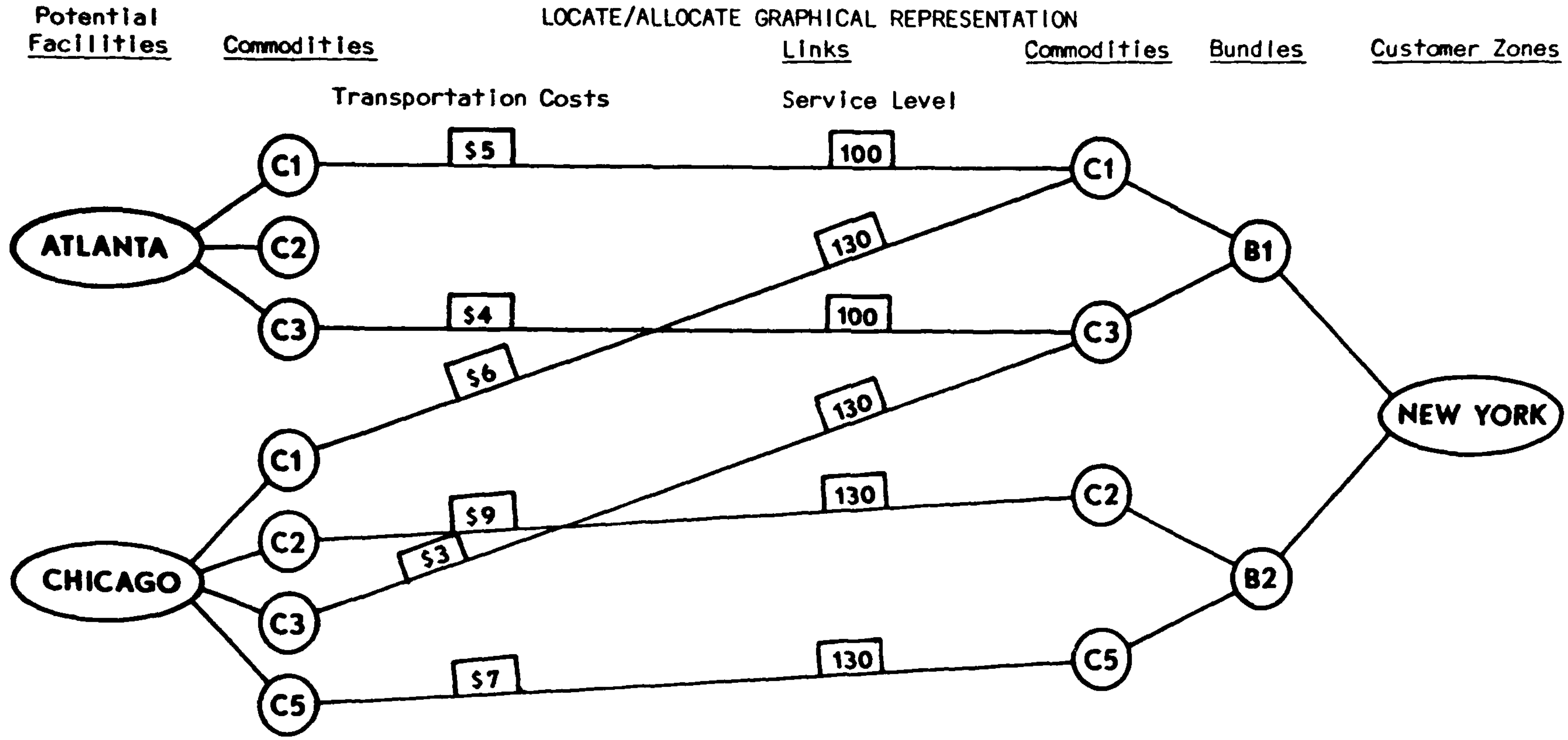
ii) the optimal allocation of customer demands to the selected facilities.

In such problems, each customer (which may in fact represent a customer zone, or aggregation of customers) requires one or more commodity bundles, where a given bundle consists of specified quantities of different commodities. Each bundle, in turn, consumes a particular amount of facility capacity, determined by the specified quantities of the commodities in the bundle. Additionally, in practical settings, it is often required that the customer receive a bundle from a single facility. (That is, the delivery of a bundle cannot be split between multiple facilities. This requirement is sometimes referred to as single-sourcing.)

Figure 2 is a graphical representation of the problem structure. Existing or potential facilities, on the left, supply commodities to customer zones (only one customer is shown). Each link from a facility/commodity node to a customer/commodity node has associated with it a cost and a service level, which may represent shipping time, mileage, or another measure of service. Often times, corporate poli-

FIGURE 2

LOCATE/ALLOCATE GRAPHICAL REPRESENTATION



cies set maximum standards on service level and on the maximum number of facilities that can be used.

This type of logistics location/allocation model gives rise to a large integer model since there is one 0-1 variable for each facility site/capacity option, and one 0-1 variable for each possible assignment of a customer bundle to a facility. Consequently, a problem with 30 facility sites/capacities and 200 customer/bundles may yield a problem with 200 x 30 + 30 = 6030 0-1 variables.

Based on the recent integer programming breakthroughs for solving generalized assignment problems [39] and the fact that logistics location/allocation problems can be modeled as a close variant of the generalized assignment problem, an efficient computer system [1] has been built and successfully applied as illustrated in the following.

### 5.1 A FOOD PROCESSOR'S APPLICATION

A major U.S. food processor and distributor has used the location/allocation system called LOCATE/ALLOCATE [1]. In this application, the food processor was using approximately 20 warehouses to distribute its products across the country. It was very uncertain whether the warehouses were in the right locations and whether they were too few or too many. Several of the key warehouses had severely limited throughput capacity. Using LOCATE/ALLOCATE about 17 potential sites, in addition to the existing 20, were evaluated to determine the optimal number of location of warehouses. At the same time, the system determined the customer zones each warehouse should serve for each of 6 commodity/freight mode combinations. A very fine resolution of demand was obtained using 185 customer zone/bundles to represent the continental U.S. This gave rise to an integer programming

problem with 1148 constraints and 9454, 0-1 variables. In several sessions with LOCATE/ALLOCATE approximately 30 runs were made varying the capacities of the sites, forcing certain sites to be used, etc. The time required to optimize each case ranged from 5 seconds to 3 minutes on an Amdahl 470 V-6.

After the optimal configuration was found, each implied change from the existing distribution network was tested to determine its contribution to improved profits. The result was a set of final recommendations identifying a small set of specific changes to the distribution network capable of producing major savings.

### 5.2 A COMBINED APPLICATION

Ciba-Geigy, Inc. has undertaken to integrate the treatment of two major facets of its operations, one using a model of the type solved by LOCATE/ALLOCATE and the other using a dynamic production, distribution, and inventory model somewhat similar to the basic Agrico model. The two models work synergistically, each enhancing the function of the other. LOCATE/ALLOCATE determines optimal long-term field warehouse locations and customer assignments to warehouses. The dynamic model then uses these warehouse locations, as well as the specific demand allocated to each warehouse by LOCATE/ALLOCATE, to do optimal tactical planning over multiple periods.

## 6. CONCLUSION

Major advances in recent years have led to network and integer programming algorithms of greatly increased efficiency. These advances have, in several cases, already been incorporated in new models

and modeling techniques, and applied to logistics problems in many industries. The enhanced solution power has led to:

- new problem formulations
- more detailed models and solutions
- new inter-departmental coordinating capability not usually associated with models
- a new ability to use full-scale logistics models in an interactive mode, with all the advantages associated with hands-on use of a model.

Recent applications of logistics modeling of the type cited are changing the way physical distribution and production planners do their jobs, and have resulted in millions of dollars of documented cost savings and smoother running operations.

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Recent developments in mathematical optimization are substantially enhancing the scope and power of logistics planning systems. Based on these advances, successful applications of sophisticated mathematical optimization logistics systems are occurring worldwide. This paper briefly discusses some of these applications and advances.

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